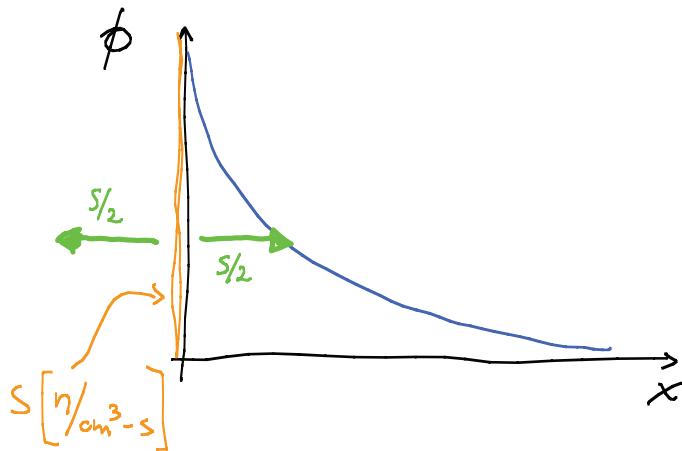


3) Okrajové úlohy difúze neutronů bez štěpení v nekonečně vrstevném prostředí

Nový nápis

11.3.2009

Nekonečné prostředí



• Balance neutronů

• Fickův zákon

• $\phi < \infty$

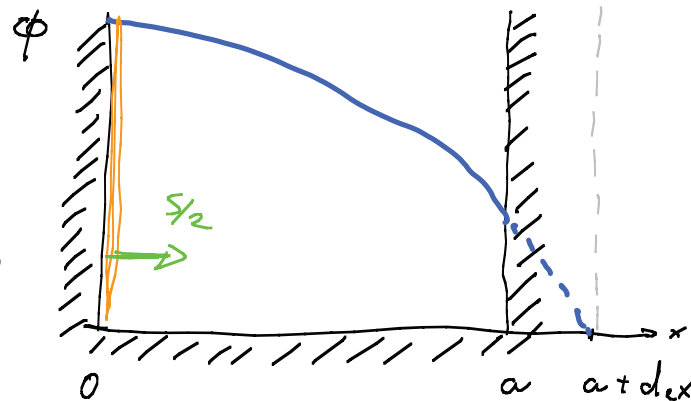
• $x > 0$: $Q(x) = S\delta(x) \leadsto \lim_{x \rightarrow 0^+} J(x) = \frac{S}{2}$

$$\Rightarrow \phi(x) = -\frac{SL}{2D} e^{-x/L}$$

Konečné prostředí

• okrajové podmínky, např.:

$$\phi(a+d_{\text{ex}}) \equiv \phi(a_{\text{ex}}) \approx \phi(a) = 0$$



$x > 0$:

$$\frac{d^2 \phi(x)}{dx^2} - \frac{1}{L^2} \phi(x) = 0, \quad L = \sqrt{\frac{D}{\Sigma_m}}$$

- obecné řešení: $\phi(x) = A e^{-x/L} + B e^{x/L}$

- O.P. vpravo: $0 = \phi(a) = A e^{-a/L} + B e^{a/L} \quad | \cdot e^{-a/L} \Rightarrow B = -A e^{-2a/L}$

$$\phi(x) = A \left[e^{-x/L} - e^{(x-2a)/L} \right]$$

- zdrojová podmínka: $\lim_{x \rightarrow 0^+} J(x) = \frac{S}{2}$ Fickův zákon: $J(x) = -D \frac{d\phi(x)}{dx} = \frac{AD}{L} \left[e^{-x/L} + e^{(x-2a)/L} \right]$

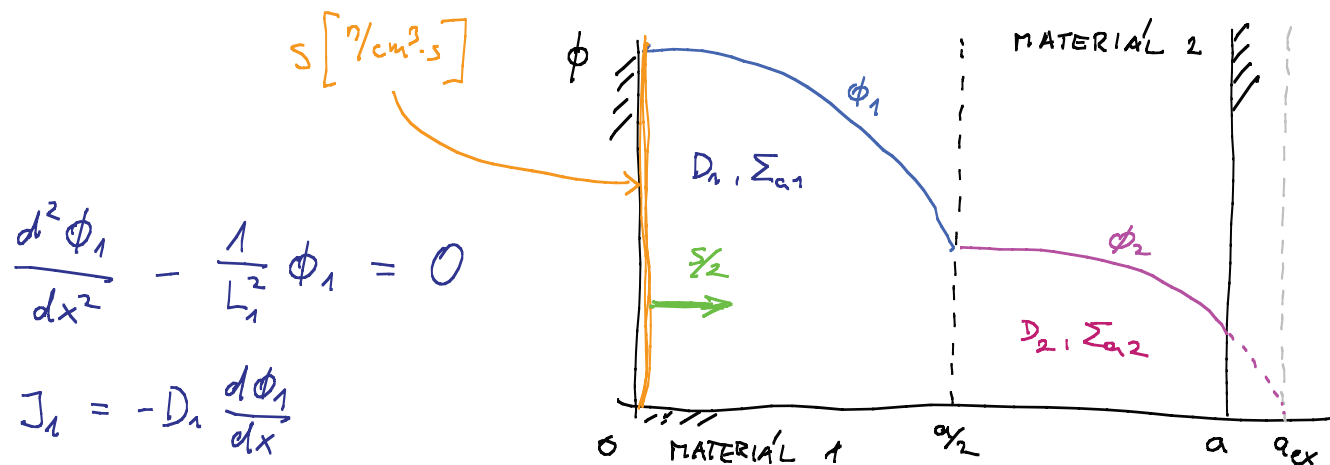
(O.P. vlevo)

$$\frac{S}{2} = \lim_{x \rightarrow 0^+} J(x) = \frac{AD}{L} (1 + e^{-2a/L})$$

$$\Rightarrow A = \frac{SL}{2D} \cdot \frac{1}{1 + e^{-2a/L}} \Rightarrow \phi(x) = \frac{SL}{2D} \cdot \frac{e^{-x/L} - e^{(x-2a)/L}}{1 + e^{-2a/L}} \quad | \cdot \frac{e^{a/L}}{e^{a/L}}$$

$$\boxed{\phi(x) = \frac{SL}{2D} \frac{\sinh\left(\frac{a-x}{L}\right)}{\cosh\left(\frac{a}{L}\right)}}$$

Prostředí s různými materiály



$$\frac{d^2 \phi_1}{dx^2} - \frac{1}{L_1^2} \phi_1 = 0$$

$$J_1 = -D_1 \frac{d\phi_1}{dx}$$

$$\frac{d^2 \phi_2}{dx^2} - \frac{1}{L_2^2} \phi_2 = 0$$

$$J_2 = -D_2 \frac{d\phi_2}{dx}$$

$$\textcircled{1} \lim_{x \rightarrow 0^+} J_1(x) = \frac{S}{2}$$

② spojitost teploty na rozhraní

$$\phi_1(a/2) = \phi_2(a/2)$$

③ spojitost proudu na rozhraní

$$J_1(a/2) = J_2(a/2)$$

$$\textcircled{4} \phi_2(a + d_{ex}) = 0$$

$$\phi_1(x) = \overset{\textcircled{1}}{A_1} \cosh \frac{x}{L_1} + \overset{\textcircled{2}}{B_1} \sinh \frac{x}{L_1}$$

$$J_1(x) = -\frac{D_1 A_1}{L_1} \sinh \frac{x}{L_1} - \frac{D_1 B_1}{L_1} \cosh \frac{x}{L_1}$$

$$\phi_2(x) = \overset{\textcircled{3}}{A_2} \cosh \frac{x}{L_2} + \overset{\textcircled{4}}{B_2} \sinh \frac{x}{L_2}$$

$$J_2(x) = -\frac{D_2 A_2}{L_2} \sinh \frac{x}{L_2} - \frac{D_2 B_2}{L_2} \cosh \frac{x}{L_2}$$

4 neznáme
4 podmienky

$$\textcircled{1} \quad \frac{S}{2} = \lim_{x \rightarrow 0^+} \left(-\frac{D_1 A_1}{L_1} \sinh \frac{x}{L_1} - \frac{D_1 B_1}{L_1} \cosh \frac{x}{L_1} \right) = -\frac{D_1 B_1}{L_1}$$

$$\Rightarrow B_1 = -\frac{SL_1}{2D_1}$$

$$\Phi_1(x) = A_1 \cosh \frac{x}{L_1} - \frac{SL_1}{2D_1} \sinh \frac{x}{L_1}$$

$$J_1(x) = -\frac{D_1 A_1}{L_1} \sinh \frac{x}{L_1} + \frac{S}{2} \cosh \frac{x}{L_1}$$

$$\textcircled{4} \quad 0 = \Phi_2(a_{ex}) = A_2 \cosh \frac{a_{ex}}{L_2} + B_2 \sinh \frac{a_{ex}}{L_2}$$

$$\Rightarrow A_2 = -B_2 \tanh \frac{a_{ex}}{L_2}$$

$$\Phi_2(x) = B_2 \left(\sinh \frac{x}{L_2} - \tanh \frac{a_{ex}}{L_2} \cosh \frac{x}{L_2} \right)$$

$$J_2(x) = \frac{D_2 B_2}{L_2} \left(\tanh \frac{a_{ex}}{L_2} \sinh \frac{x}{L_2} - \cosh \frac{x}{L_2} \right)$$

$$\textcircled{2} \quad \Phi_1(a/2) - \Phi_2(a/2) = 0 \quad \textcircled{3} \quad J_1(a/2) - J_2(a/2) = 0$$

$$\begin{bmatrix} \cosh \frac{a}{2L_1} & \tanh \frac{a_{ex}}{L_2} \cosh \frac{a}{2L_2} - \sinh \frac{a}{2L_2} \\ -\frac{D_1}{L_1} \sinh \frac{a}{2L_1} & \frac{D_2}{L_2} \cosh \frac{a}{2L_2} - \frac{D_2}{L_2} \tanh \frac{a_{ex}}{L_2} \sinh \frac{a}{2L_2} \end{bmatrix} \begin{bmatrix} A_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} \frac{SL_1}{2D_1} \sinh \frac{a}{2L_1} \\ -\frac{S}{2} \cosh \frac{a}{2L_1} \end{bmatrix}$$

$$\text{In[1]:= } \mathbf{M} = \left(\begin{array}{c|c} \text{Cosh}\left[\frac{a}{2 L_1}\right] & \text{Tanh}\left[\frac{a_{\text{ex}}}{L_2}\right] \text{Cosh}\left[\frac{a}{2 L_2}\right] - \text{Sinh}\left[\frac{a}{2 L_2}\right] \\ \hline \frac{-D_1}{L_1} \text{Sinh}\left[\frac{a}{2 L_1}\right] & \frac{D_2}{L_2} \text{Cosh}\left[\frac{a}{2 L_2}\right] - \frac{D_2}{L_2} \text{Tanh}\left[\frac{a_{\text{ex}}}{L_2}\right] \text{Sinh}\left[\frac{a}{2 L_2}\right] \end{array} \right);$$

$$\mathbf{b} = \left\{ \frac{S L_1}{2 D_1} \text{Sinh}\left[\frac{a}{2 L_1}\right], -\frac{S}{2} \text{Cosh}\left[\frac{a}{2 L_1}\right] \right\};$$

$$\text{In[3]:= } \{\mathbf{A}_1, \mathbf{B}_2\} = \text{LinearSolve}[\mathbf{M}, \mathbf{b}] // \text{FullSimplify}$$

$$\text{Out[3]= } \left\{ \left(S L_1 \left(\text{Cosh}\left[\frac{a - 2 a_{\text{ex}}}{2 L_2}\right] \text{Sinh}\left[\frac{a}{2 L_1}\right] D_2 L_1 - \text{Cosh}\left[\frac{a}{2 L_1}\right] \text{Sinh}\left[\frac{a - 2 a_{\text{ex}}}{2 L_2}\right] D_1 L_2 \right) \right) / \right. \\ \left(2 D_1 \left(\text{Cosh}\left[\frac{a}{2 L_1}\right] \text{Cosh}\left[\frac{a - 2 a_{\text{ex}}}{2 L_2}\right] D_2 L_1 - \text{Sinh}\left[\frac{a}{2 L_1}\right] \text{Sinh}\left[\frac{a - 2 a_{\text{ex}}}{2 L_2}\right] D_1 L_2 \right) \right), \\ \left. - \left(S \text{Cosh}\left[\frac{a_{\text{ex}}}{L_2}\right] L_1 L_2 \right) / \left(2 \text{Cosh}\left[\frac{a}{2 L_1}\right] \text{Cosh}\left[\frac{a - 2 a_{\text{ex}}}{2 L_2}\right] D_2 L_1 - 2 \text{Sinh}\left[\frac{a}{2 L_1}\right] \text{Sinh}\left[\frac{a - 2 a_{\text{ex}}}{2 L_2}\right] D_1 L_2 \right) \right\}$$

$$\text{In[4]:= } \{\mathbf{A}_2, \mathbf{B}_1\} = \left\{ -\mathbf{B}_2 \text{Tanh}\left[\frac{a_{\text{ex}}}{L_2}\right], -\frac{S L_1}{2 D_1} \right\} // \text{FullSimplify}$$

$$\text{Out[4]= } \left\{ \frac{S \text{Sinh}\left[\frac{a_{\text{ex}}}{L_2}\right] L_1 L_2}{2 \text{Cosh}\left[\frac{a}{2 L_1}\right] \text{Cosh}\left[\frac{a - 2 a_{\text{ex}}}{2 L_2}\right] D_2 L_1 - 2 \text{Sinh}\left[\frac{a}{2 L_1}\right] \text{Sinh}\left[\frac{a - 2 a_{\text{ex}}}{2 L_2}\right] D_1 L_2}, -\frac{S L_1}{2 D_1} \right\}$$

■ Předpokládaný průběh neutronového toku

$$\text{In}[5]:= \phi_1[x] = A_1 \cosh\left[\frac{x}{L_1}\right] + B_1 \sinh\left[\frac{x}{L_1}\right];$$

$$\phi_2[x] = A_2 \cosh\left[\frac{x}{L_2}\right] + B_2 \sinh\left[\frac{x}{L_2}\right];$$

■ Ověření předepsaných podmínek

$$\text{In}[7]:= \phi_2[a_{\text{ex}}]$$

$$\text{Out}[7]= 0$$

$$\text{In}[8]:= \text{Limit}[-D_1 \partial_x \phi_1[x], x \rightarrow 0]$$

$$\text{Out}[8]= \frac{S}{2}$$

$$\text{In}[10]:= \phi_1\left[\frac{a}{2}\right] - \phi_2\left[\frac{a}{2}\right] // \text{FullSimplify}$$

$$\text{Out}[10]= 0$$

$$\text{In}[12]:= D_1 \partial_x \phi_1[x] - D_2 \partial_x \phi_2[x] /. x \rightarrow \frac{a}{2} // \text{FullSimplify}$$

$$\text{Out}[12]= 0$$

■ Ukázkový příklad

$$\text{In}[62]:= D_1 = 4; L_1 = 5; D_2 = 0.4; L_2 = 3; a = 20; a_{\text{ex}} = a + 2 D_2; S = 2;$$

$$\text{In}[63]:= \text{Show}\left[\text{Plot}\left[\phi_1[x], \{x, 0, \frac{a}{2}\}\right], \text{Plot}\left[\phi_2[x], \{x, \frac{a}{2}, a_{\text{ex}}\}\right], \text{PlotRange} \rightarrow \text{Automatic}\right]$$

