Introduction to quantum turbulence

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The term quantum turbulence denotes the turbulent motion of quantum fluids, systems such as superfluid helium and atomic Bose–Einstein condensates, which are characterized by quantized vorticity, superfluidity, and, at finite temperatures, two-fluid behavior. This article introduces their basic properties, describes types and regimes of turbulence that have been observed, and highlights similarities and differences between quantum turbulence and classical turbulence in ordinary fluids. Our aim is also to link together the articles of this special issue and to provide a perspective of the future development of a subject that contains aspects of fluid mechanics, atomic physics, condensed matter, and low-temperature physics.

quantized vortices | absolute zero | Kolmogorov spectrum

Turbulence is a spatially and temporally complex state of fluid motion. Five centuries ago, Leonardo da Vinci noticed that water falling into a pond creates eddies of motion. Today, turbulence still provides physicists, applied mathematicians, and engineers with a continuing challenge. Leonardo realized that the motion of water shapes the landscape. Today's researchers appreciate that many physical processes, from the generation of the Galactic magnetic field to the efficiency of jet engines, depend on turbulence.

The articles in this collection are devoted to a special form of turbulence known as quantum turbulence (1–3), which appears in quantum fluids. Quantum fluids differ from ordinary fluids in three respects: (*i*) they exhibit two-fluid behavior at nonzero temperature or in the presence of impurities, (*ii*) they can flow freely, without the dissipative effect of viscous forces, and (*iii*) their local rotation is constrained to discrete vortex lines of known strength (unlike the eddies in ordinary fluids, which are continuous and can have arbitrary size, shape, and strength). Superfluidity and quantized vorticity are extraordinary manifestations of quantum mechanics at macroscopic-length scales.

Recent experiments have highlighted quantitative connections, as well as fundamental differences, between turbulence in quantum fluids and turbulence in ordinary fluids (classical turbulence). The relation between the two forms of turbulence is indeed a common theme in the articles collected here. Because different scientific communities (low-temperature physics, condensed-matter physics, fluid dynamics, and atomic physics) have contributed to progress in quantum turbulence,[†] the aim of this article is to introduce the main ideas in a coherent way.

Quantum Fluids

In this series of articles, we shall be concerned almost exclusively with superfluid ^{4}He, the B-phase of superfluid ^{3}He, and, to a lesser extent, with ultracold atomic gases. These systems exist as fluids at temperatures on the order of a Kelvin, milliKelvin, and microKelvin, respectively.[‡] Their constituents are either bosons (such as ^{4}He atoms with zero spin) or fermions (such as ^{3}He atoms with spin 1/2). This difference is fundamental: the former obey Bose–Einstein statistics and the latter Fermi–Dirac quantum statistics.

Let us consider an ideal (noninteracting) gas of bosons first. Under normal conditions at room temperature, the de Broglie wavelength λ of each atom is much smaller than the average separation d between the atoms; if the temperature T is lowered, λ increases until, if T is sufficiently small, λ becomes larger than d, and the quantum–mechanical wave aspects become dominant. The resulting phase transition, called the Bose–Einstein condensation (5), is characterized by a macroscopic number of bosons occupying the state of zero energy. Although the possibility of Bose–Einstein condensation was raised in 1924–1925, its direct experimental demonstration in dilute ultracold atomic gases occurred only in 1995. To achieve superfluidity (flow without friction), another ingredient is necessary: The particles must interact with each other.

In fermionic systems, at temperatures much lower than a characteristic Fermi temperature, particles occupy the interior of the Fermi sphere in the momentum space, with only relatively few particle–hole pairs, called excitations. Attractive interaction between fermions leads to an instability and the formation of Cooper pairs, which are bosons undergoing Bose condensation, resulting either in superconductivity in charged systems of electrons in crystal lattices or in superfluidity in systems consisting of neutral atoms. The outcome is surprisingly similar to what happens for bosons: Superfluidity arises from the formation of a coherent particle field that can be described using the formalism of the order parameter or a condensate macroscopic wave function.

Superfluidity of ^{4}He was experimentally discovered by Kapitza and by Allen and Misener in 1938 although it is now believed that Kamerlingh Onnes must have had superfluid helium in his apparatus when he first liquefied helium in Leiden in 1908. He and other pioneers of low-temperature physics soon discovered that, below a critical temperature $T_\lambda \cong 2.17$ K, liquid helium displays unusual behavior. They therefore called it helium I and helium II, respectively, above and below this temperature. Still, in 1938, London linked the properties of helium II to Bose–Einstein condensation. Further progress in understanding superfluidity of ^{4}He was driven by the work of Landau based on different considerations.

The physical properties of normal liquid ^{3}He at milliKelvin temperatures are well described in the frame of the Fermi liquid theory

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[†]The term quantum turbulence was introduced into the literature in 1986 by R. J. Donnelly in a symposium dedicated to the memory of G. I. Taylor (4).

[‡]What determines the need for a quantum mechanical description is not the absolute value of temperature but whether it is lower than a certain characteristic temperature of the system (for example, the Fermi temperature); instances of quantum fluids at high temperature are exciton-polariton condensates (≈ 300 K) and neutron stars (10^6 to 10^9 K).

of Landau. Note the striking difference in kinematic viscosities of ^{4}He (the lowest of all known fluids, two orders of magnitude less than water's) and of ^{3}He near the superfluid transition (comparable with that of air or olive oil). Superfluidity of ^{3}He was theoretically predicted by Pitaevskii and experimentally discovered by Osheroff, Richardson, and Lee in 1973. Cooper pairs consisting of two ^{3}He atoms (which themselves are fermions), rotating about their center of mass, are bosons of total spin and orbital numbers equal to one. This property allows three different projections on quantization axes in orbital and spin spaces, and, as a consequence, several different superfluid phases of ^{3}He exist. The classification of them and of the types of quantized vorticity in ^{3}He is beyond the scope of this article (in particular, because quantum turbulence has been studied only in the B phase).

Finally, recently developed experimental methods of laser and evaporative cooling have opened up a new road to ultralow-temperature physics: MicroKelvin clouds of dilute atoms were generated and nanoKelvin temperatures achieved to explore quantum-degenerate gases, providing additional working fluids to study quantum turbulence.

The two-fluid model, introduced in the context of ^{4}He first by Tizsa and (based on different considerations) by Landau, is a convenient level of description of quantum turbulence. Below T_λ, ^{4}He is described as a viscous normal fluid (a gas of thermal excitations, called phonons and rotons, that carry the entire entropy content) coexisting with an inviscid superfluid (related but not equal to the condensate fraction). The density of helium II, essentially temperature-independent, can be decomposed into $\rho = \rho_n + \rho_s$, where the normal fluid's and superfluid's densities, ρ_n and ρ_s, respectively, are strongly temperature-dependent: in the low-temperature limit ($T \to 0$), helium is entirely superfluid ($\rho_s/\rho \to 1$, $\rho_n/\rho \to 0$) whereas, in the high temperature limit ($T \to T_\lambda$), superfluidity vanishes ($\rho_s/\rho \to 0$, $\rho_n/\rho \to 1$). At temperatures below 1 K (where $\rho_n/\rho = 0.07$), in the absence of ^{3}He impurities, ^{4}He can be considered more or less a pure superfluid. Similar considerations apply for superfluid ^{3}He; its B phase can be considered a pure superfluid below about 200 μK.

The normal fluid and the superfluid support two independent velocity fields $\mathbf{v}_n$ and $\mathbf{v}_s$, respectively, and the superfluid component flows without viscous dissipation. Based on the form of the dispersion relation, Landau predicted that the superfluidity of ^{4}He disappears at flow velocities exceeding a critical value of about 60 m/s (due to the emission of quasiparticles called rotons). In a more general sense, the Landau criterion applies to any superfluid: On exceeding certain critical velocity (which in fermionic superfluids is called the Landau pair-braking velocity), it becomes energetically favorable to generate quasiparticles, which means the onset of dissipation.

What makes superfluid hydrodynamics particularly interesting is that the circulation integral $\oint_C \mathbf{v}_s \cdot \mathbf{dr}$ is equal either to the quantum of circulation $\kappa = h/m$ or to zero, depending on whether or not the integration path C encloses a quantized vortex line; here, h is Planck's constant and m the mass of the relevant boson (one atom in ^{4}He, a Cooper pair in ^{3}He-B). This quantization condition, suggested by Onsager and experimentally confirmed by Vinen, arises from the existence and the single-valuedness of a complex, macroscopic superfluid wave function ψ, and the usual quantum mechanical prescription that the velocity is proportional to the gradient of the phase of ψ. As a consequence, the superflow is not only inviscid (like the ideal Euler fluid), but also potential ($\nabla \times \mathbf{v}_s = 0$). Vortex lines can be viewed as holes with circulation. Moving around the vortex axis, the phase of ψ changes by 2π (multiple values of κ are unstable in helium II), corresponding to a persistent azimuthal superfluid velocity of the form $v_s = \kappa/(2\pi r)$ where r is the radial distance from the axis. An isolated vortex line is thus a stable topological defect. On its axis, real and imaginary parts of ψ vanish; the narrow region where the density drops from its value at infinity to zero is proportional to the healing (or coherence) length ξ, which depends on the strength of the interaction between the bosons. In ^{4}He, $\xi \approx 10^{-10}$m; in ^{3}He-B, ξ is about 100 times larger, and in atomic condensates even larger (1/100–1/10 of the system size).

Quantized vortex lines are nucleated intrinsically or extrinsically (that is, from already existing vortex lines, which, twisting under the influence of the superflow and then reconnecting, generate new vortex loops). Nucleation is opposed by a potential barrier, which, upon exceeding a critical velocity v_c, can be overcome either thermally or by quantum tunneling. In helium II (except close to T_λ), intrinsic nucleation requires $v_c \approx 10$m/s, large enough to make it unlikely unless induced by a fast-moving ion. Experimentally reported values of $v_c \approx 10^{-2}$m/s are associated with extrinsic nucleation from remnant vortices pinned to the walls of the container. In ^{3}He-B, both intrinsic and extrinsic nucleation are possible, and (as in atomic condensates) $v_c \approx 10^{-3}$m/s. Vortex lines can also form by the Kibble–Zurek mechanism when helium is cooled through the superfluid second-order phase transition: The phase of ψ, unable to adjust everywhere at the same time, leaves vortex lines as defects.

The possibility of quantum turbulence was first raised by Feynman (6); soon afterward, Vinen showed that a turbulent vortex tangle can be generated in the laboratory (7) by applying a heat flux to helium II. Two properties of vortex lines are important for quantum turbulence. The first is the mutual friction force (7, 8), which couples the superfluid and the normal fluid. It arises from the scattering of thermal quasiparticles (constituents of the normal fluid) by the velocity field of the vortex lines. The second property comprises Kelvin waves, which are helical displacements of the vortex core that rotate with angular velocity $\omega \sim \kappa k^2$, where k is the one-dimensional wavenumber (shorter waves rotate faster). Kelvin waves arise from the tension of the vortex lines (the kinetic energy of a circulating superfluid about a unit length of line). Their direct observation is reported in the article by Fonda et al. (9). At finite temperatures, Kelvin waves are damped by mutual friction, but, below 1 K, they propagate more freely and lead to acoustic emission at large values of k. The transfer of energy to such large k by a Kelvin wave cascade (analogous to the Kolmogorov cascade of classical turbulence) explains the observed decay of turbulence at low temperatures, as discussed in the articles by Barenghi et al. (10) and by Walmsley et al. (11); in the weak-amplitude regime, Kelvin waves can be studied using wave-turbulence theory [see the article by Kolmakov et al. (12)].

The difference between the ideal fluid and the superfluid can be better appreciated by noticing the link between superfluidity and superconductivity, and the relation between an ideal conductor and a superconductor; the latter behaves as an ideal diamagnetic substance that, below certain critical conditions, expels the externally applied magnetic field from its interior (the Meissner effect). In superfluidity, the corresponding feature is that the superflow is always curl-free, or potential, independently on whether rotating or quiescent samples were cooled through the superfluid transition. Quantized vortices exist in superconductors and may reconnect [the physical quantity that is quantized here is the magnetic flux, in units $2\pi\hbar/(2e)$, where $2e$ is the charge of two electrons constituting a Cooper pair]. The motion of vortex lines and flux tubes, however, is not the same: If displaced, the former move (almost) along the binormal and the latter (almost) along the normal direction (13), which possibly explains the absence of quantum turbulence in superconductors.

Experimental Methods for Quantum Turbulence

Some of the experimental methods used to probe turbulence in ordinary viscous fluids have been used for quantum turbulence. They include small Pitot tubes (14) to measure pressure-head fluctuations (giving access to velocity probability density distributions, structure functions, and energy spectra) and a plethora of small mechanical oscillators, such as spheres, wires, nanowires, grids, and quartz tuning forks (for recent review, see ref. 15), that can generate and detect quantum turbulence.

Direct visualization is invaluable in classical turbulence: Methods such as particle image and particle-tracking velocimetry, applied to scientific and industrial problems, give quantitative data and qualitative information such as flow patterns. Although the application of these methods to cryogenic flows is difficult for reasons that are both technical (e.g., optical access to the experimental volume) and fundamental (e.g., the presence of two velocity fields and interaction (16) of quantum vortices with particles—in most cases, μm-sized frozen flakes of solid hydrogen or deuterium), it has already led to important results on the direct observation of individual quantized vortices (17), their reconnections (18), Lagrangian velocity (19), and acceleration (20) statistics. Another visualization technique (21), based on fluorescence, employs neutral He_2^* triplet molecules as tracers.

An advantage of these conventional methods is that they allow direct comparison between classical turbulence above T_λ and quantum turbulence below T_λ, but, in the latter case, care must be taken to understand whether particles trace the motion of the normal fluid, the superfluid, or the vortex lines. The current status of the subject is described in the article by Guo et al. (22).

Second-sound attenuation is the most powerful (and historically the oldest) (7) measurement tool in ^{4}He, revealing the vortex-line density L—the total length of the quantized vortex line in a unit volume (23). Because second sound is an antiphase oscillation of normal and superfluid components, this technique cannot be used below 1 K (because there is little normal fluid) or in ^{3}He at any temperature (second-sound waves are overdamped by the large viscosity of the normal fluid).

Helium ions have been successfully used to detect quantum vortices in ^{4}He—for example, to investigate the decay of inhomogeneous quantum turbulence created by ultrasonic transducers at about 1.5K (24). An improved technique based on negative ions has been recently introduced for measurements of decaying quantum turbulence below 1 K (25, 26). Negative ions (electron bubbles) are injected by a sharp field-emission tip and manipulated by an applied electric field. Bare ions dominate for $T > 0.8$ K whereas, for $T < 0.7$ K, they become self-trapped in the core of quantized vortex rings of diameter about 1 μm (the rings are intrinsically nucleated when the ions are accelerated past v_c by an imposed electric field). Short pulses of ions or rings are sent across the experimental cell. The relative reduction of the amplitude of pulses of ions or rings detected at the collector on the opposite side of the helium cell after their interaction with quantum turbulence is converted to vortex-line density.

In ^{3}He-B, information on quantum turbulence has been obtained using NMR (27). Another experimental technique in ^{3}He-B is the Andreev scattering of quasiparticles by the velocity field of quantized vortices (28), described in the article by Fisher et al. (29).

Finally, in atomic Bose–Einstein condensates, vortices are created by stirring or shaking the trap, phase imprinting, or moving a laser "spoon" across the condensate; images are taken after releasing the trap and expanding the condensate, as explained by White et al. (30).

Theoretical Models of Quantum Turbulence

Unlike classical turbulence, studied on the solid ground of the Navier–Stokes equation, there is no single equation governing the motion for quantum turbulence, but rather a hierarchy of models at different length scales, each with its own limitations. It is as if one is unable to describe the trees and the forest in a unified way: We need one (microscopic) model that accounts for the close-up details of one tree or few trees, a second (mesoscopic) model that, from further away, does not resolve the details of the trees but still distinguishes individual trees as isolated sticks, and a third (macroscopic) model that does not resolve trees at all but recognizes where the forest is sparser or denser. In this spirit, we note that helium turbulence is characterized by a wide separation of length scales $\xi \ll \ell \ll D$, where ξ (already defined) is a measure of the vortex core, ℓ is the average distance between vortex lines (usually estimated as $\ell \approx L^{-1/2}$), and D is the size of the system: typically, $\xi \approx 10^{-10}$m in ^{4}He (10^{-8}m in ^{3}He-B), $\ell \approx 10^{-5}$m, and $D \approx 10^{-2}$m. In atomic condensates, these scales are not as widely separated: $\xi < \ell < D$.

The microscopic model is the Gross–Pitaevski equation for a Bose–Einstein condensate, obtained (after suitable approximations) from the Hamiltonian of a Bose gas undergoing two-bodies collisions. The Madelung transformation makes the hydrodynamics interpretation of the wave function ψ apparent, yielding the classical continuity equation and a modified Euler equation. It differs from the classical Euler equation because of the presence of the so-called quantum pressure, which, differentiating a superflow from a perfect Euler flow, is responsible for vortex reconnections (31), for sound pulses at reconnection events (32), and for the nucleation of vortices near a boundary (e.g., an ion) (33) or a strong density variation (e.g., cavitation) (24, 34).

The Gross–Pitaevskii equation has been used to study turbulence in atomic condensates (35, 36), but its application to superfluid helium is only qualitative. Its dispersion relation does not exhibit the roton minimum and is valid only near $T = 0$. For generalizations to finite temperature, we refer the reader to the article of Berloff et al. (37). One approach worth mentioning is the Zaremba–Nikuni–Griffin formalism (38), which couples the Gross–Pitaevskii equation to a Boltzmann equation for the thermal cloud of noncondensed atoms, allowing atomic collisions within the thermal cloud and between thermal cloud and condensate: The outcome of this self-consistent interaction is the emergence (39) of dissipative effects on vortex motion (mutual friction).

A mesoscale approach that shuns effects at the scale of ξ is the vortex-filament model of Schwarz (40), which represents vortex lines as space curves of infinitesimal thickness and circulation κ. At $T = 0$, a point on a vortex line moves with the total superfluid velocity at that point—which is the self-induced velocity calculated using the Biot–Savart law, plus any externally imposed superflow. For $T > 0$, the motion results from the balance of Magnus and friction forces. Schwarz's insight was to recognize that, to describe quantum turbulence, his equation of motion must be supplemented with an algorithmic procedure to reconnect vortex lines that approach each other by a distance less than the discretization distance along the lines (thus moving away from the realm of Euler dynamics).

The vortex-filament model is perhaps the most useful and flexible numerical tool for quantum turbulence in ^{4}He and ^{3}He-B; the state of the art is described by Baggaley and Hänninen (41). It is therefore important to appreciate its limitations. The first is that (unlike the Gross–Pitaevskii equation) it does not describe acoustic losses of energy by rapidly rotating Kelvin waves at very low temperature. The second limitation is that the computational cost of the Biot–Savart law grows rapidly as N^2 (where N is the number of discretization points N along the vortex lines). To speed up his calculations, Schwarz replaced the Biot–Savart law with its local induction approximation, which neglects any vortex interaction and requires an arbitrary mixing step to achieve a statistically steady state of turbulence (42)—an approximation that is thought as unsatisfactory by todays's standards. This problem was recently solved (43, 44) by adapting to vortex dynamics the $N \log N$ tree-algorithm created for computational astrophysics (45). The third difficulty of Schwarz's model is that (with notable exceptions) (46, 47) the normal fluid velocity is imposed rather than computed self-consistently by solving the Navier–Stokes equation (suitably modified by a friction term): again, the reason is the computational cost involved.

The problem of self-consistency is solved, at a more macroscopic level, by the Hall–Vinen–Bekharevich–Khalatnikhov (HVBK) equations, originally developed for rotating helium. The HVBK equations describe the motion of fluid parcels containing a large number of parallel vortex lines. Coarse graining allows treating superfluid vorticity as a continuous classical field, generalizing the original two-fluid equations of Landau. The HVBK equations successfully predict the oscillation of a rotating vortex lattice, the Glaberson instability, the instability of helium Couette flow, and the transition to Taylor vortices (48, 49), flows for which the assumption is valid that the vortex lines are locally aligned within each fluid parcel. Application of the HVBK equations to turbulence is not justified for randomly oriented vortex lines, as the net superfluid vorticity in each fluid parcel would be zero, yielding zero friction, despite the nonzero vortex-line density. Modifications of the HVBK equations have been developed, neglecting the vortex tension and approximating the mutual friction (50, 51). Such models probably underestimate friction dissipation, but the coupled motion of both fluids is computed self-consistently. Shell models of turbulence (52, 53) and Leith models (54) are variants of the HVBK equations trading spatial information for that in the k-space.

Types and Regimes of Quantum Turbulence

It is useful to classify the various types of turbulent flows that can be generated in a quantum fluid, keeping in mind that more classification schemes might well be possible. To start with, the strong temperature dependence of superfluid and normal fluid components and the relatively high kinematic viscosity of ^{3}He-B compared with ^{4}He suggest the following natural distinction:

i) Pure quantum (superfluid) turbulence in low-temperature ^{4}He and ^{3}He-B. This is conceptually the simplest (but experimentally the most challenging) form of quantum turbulence: a single turbulent superfluid (the normal fluid being absent or negligible). This prototype of turbulence—a tangle of quantized vortex line—can be excited at small length scales by injecting ions or vortex rings (26), or at larger length scales using vibrating objects (15, 28), or by suddenly halting the rotation and destabilizing an existing vortex lattice (25). Besides the residual friction potentially caused by thermal excitations, dissipation of kinetic energy is possible due to acoustic emission from short and rapidly rotating Kelvin waves (55) and from vortex reconnections (31). The length scale required for efficient acoustic emission is much shorter than the typical curvature at the quantum length scale $\ell \approx L^{-1/2}$, but can be achieved by a Kelvin wave cascade—which is the energy transfer to increasingly smaller scales arising from the nonlinear interaction of Kelvin waves (56, 57); this mechanism is discussed by Barenghi et al. (10)). In ^{3}He-B, the larger vortex core limits the wavenumber range of the Kelvin wave cascade, but Caroli–Matricon bound states in the vortex core provide an additional dissipation mechanism (58).

ii) Quantum turbulence with friction in finite-temperature ^{3}He-B. The main feature of this form is that the highly viscous normal fluid is effectively clamped to the walls. The mutual friction force acts on all length scales and affects the dynamics of quantized vortices, damping the energy of Kelvin waves into the normal fluid. The role of friction increases with rising temperature to the point that, upon exceeding a critical temperature, turbulence can be suppressed altogether. Temperature thus plays an analogous role to that of the Reynolds number in classical turbulence (27). This type of quantum turbulence is discussed in the article by Eltsov et al. (59).

iii) Two coupled turbulent fluids in ^{4}He. This type of turbulence is easily generated by stirring helium II by mechanical means [e.g., towed grids (60–62) and propellers (14)], by ultrasound (24), or by forcing it using grids and flows past bluff bodies in wind tunnels (63). Both superfluid and normal fluid component are turbulent. Because of its double nature, this is the most general and challenging type of quantum turbulence, generally richer than classical turbulence in conventional viscous fluids, presenting us with two coupled turbulent systems, one in which the vorticity is continuous and the other with quantized vortex lines. The mutual friction transfers energy from one fluid to the other so that it can act both as a source and a sink of energy for each fluid. Moreover, by combining thermal and mechanical drives, special types of turbulence can be generated in which the mean superfluid and normal fluid velocities are not necessarily the same. For example, thermal counterflow is induced by applying a voltage to a resistor (heater) located at the closed end of a channel that is open to a superfluid ^{4}He bath at the other end. The heat flux is carried away from the heater by the normal fluid alone, and, by conservation of mass, a superfluid current occurs in the opposite direction. In this way a relative (counterflow) velocity is created along the channel that is proportional to the applied heat flux that is quickly accompanied by a tangle of vortex lines (7, 64). The normal fluid is likely to be laminar for small heat fluxes and probably turbulent for large heat fluxes. Another special case is pure superflow (23), generated both mechanically and thermally in a channel whose one end or both ends are covered by superleaks (walls with holes so tiny that they are permeable only to the superfluid component).

A second possible classification of quantum turbulence is based on the form of the energy spectrum $E(k)$—the distribution of kinetic energy over the wavenumbers k (inverse length scales). Two limiting regimes have been tentatively identified:

i) Vinen or ultra-quantum turbulence. This is a random vortex tangle with a single dominant length scale, ℓ. It has long been argued (7) that steady counterflow turbulence at nonzero temperature in ^{4}He is in this regime although the energy spectrum has never been directly measured experimentally. A recent numerical calculation (65) of counterflow turbulence driven by a uniform normal flow has shown a broad energy spectrum around $k \approx 1/\ell$, confirming this state. At very low temperatures, ultraquantum turbulence has been produced in ^{4}He by ion injection (26). The main experimental evidence (26, 66) is that, if the vortex tangle is left to decay, the vortex line density decreases as $L \sim t^{-1}$, in agreement with a phenomenological model of Vinen (7), which indeed assumes a random, homogeneous, and isotropic vortex configuration. The same $L \sim t^{-1}$ decay has been observed in numerical simulations (67), which also confirmed that the energy spectrum remains broadly concentrated near $k \approx 1/\ell$.

ii) Kolmogorov or semiclassical turbulence. This regime is quite similar to classical turbulence, as the energy spectrum contains an inertial range and closely displays the celebrated K41 scaling $E(k) \sim k^{-5/3}$ over $1/D \ll k \ll /\ell$ (thus, most of the energy resides at the largest length scales). Direct evidence of Kolmogorov scaling is provided by experiments at high and intermediate temperatures (14, 63) and numerical simulations (65) in which the vortex tangle is driven by a turbulent normal fluid. At very low temperatures, the experimental evidence (26) is based on the observed decay $L \sim t^{-3/2}$, which, it has been argued (1, 66, 68), is consistent with the Kolmogorov spectrum. At $T = 0$, numerical simulations based on both the vortex-filament model (68–70) and the Gross–Pitaevskii equation (71, 72) have produced spectra consistent with the $k^{-5/3}$ scaling. Further numerical studies have revealed that the Kolmogorov energy spectrum is associated with the presence of metastable bundles of polarized quantized vortices (65, 70, 73). This insight opens the possibility of stretching such bundles (stretching of individual quantum vortices is not possible because of the quantization condition). The polarization of (part of) the vortex tangle is also discussed in the article of Vinen and Skrbek (74) on turbulence generated by oscillating objects; its importance lies in the fact that, in classical turbulence, vortex stretching is thought to be responsible for the dissipationless transfer of energy from large to small scales (energy cascade).

An important issue is the normal fluid's profile in various types of channel and pipe flows of helium II. For example, in numerical simulations of counterflow turbulence driven by uniform normal fluid, the energy spectrum broadly peaks at the mesoscales $k \approx 1/\ell$ (65) (as in ultraquantum turbulence), but the experimentally observed decay is $L \sim t^{-3/2}$ (64) (typical of quasiclassical turbulence). Thus, either large-scale structures already exist in steady-state counterflow or are generated by halting the normal fluid.

A third classification is suggested by the relative magnitude of ξ, ℓ, and D. Quantum turbulence in atomic Bose–Einstein condensates lacks the wide separation of scales typical of liquid helium: The size of typical condensates is only 10–100 times the healing length. Will the known scaling laws of turbulence manifest themselves as the size of the condensate increases? The close distance between vortices suggests that reconnections play a more important role in the kinetic energy dissipation than in helium; moreover, vortex energy can be transformed into surface oscillations of the condensate. Atomic condensates can be used to explore the crossover from 2D to 3D turbulence, offer greater flexibility than helium, as physical parameters can be engineered and vortices can be individually manipulated, and, thanks to the weak interactions, are a testing ground for the theory. Two-components (75) and spinor condensates (76) are rich new systems for turbulence. Dipolar condensates (77) may open the possibility of turbulence with unusual vortex interaction. These opportunities are discussed in the article by White et al. (30).

Outlook

Quantum turbulence is a relatively young field of research compared with conventional turbulence in viscous fluids, which has slowly but steadily progressed over several centuries. The early works on quantum turbulence (7) were mainly concerned with counterflow as a problem of heat transfer unique of liquid helium II. It was only after the seminal experiments of Donnelly, Tabeling, and coworkers (14, 61-62) that the attention shifted to concepts such as energy spectrum and vorticity decay, which are typical of the fluid dynamics literature. These and other experiments showed that, over length scales much larger than the mean intervortex distance ℓ, quantum turbulence mimics (66, 68) the properties of classical turbulence, hinting (in the spirit of Bohr's old quantum theory) that many quanta of circulation yield classical behavior. This result, together with the very low kinematic viscosity of ^{4}He, suggests that quantum turbulence can be used to study classical problems such as the temporal decay of homogeneous, isotropic turbulence or the long-standing puzzle of the Loitsianskii invariant. In general, highly turbulent flows are needed to tackle these problems. It is not difficult to generate such flows with liquid ^{4}He, and the huge-capacity liquefiers of the European Organization for Nuclear Research (CERN) are already considered for the purpose within the European project European High-Performance Infrastructures in Turbulence (EuHIT), in the frame of the seventh European Union initiative. A more challenging task is the development of miniature special probes capable of probing quantum turbulence, resolving all scales including quantum scales smaller than ℓ.

On the other hand, the existence of the ultraquantum regime is a warning signal that not all quantum turbulent flows are related to classical flows. The various types and regimes of quantum turbulence that we have identified provide a rich range of problems that we should solve using hydrodynamics models (the spirit is similar to how the known planetary atmospheres are explained by the same physical principles under different parameters).

Problems that seem particularly challenging involve either two turbulent cascades taking place in the same fluid in different regions of k-space (the Kolmogorov cascade and the Kelvin waves cascade) or two active turbulent superfluids affecting each other [e.g., two-component cold gases (75) and, when experimentally realized (78), ^{3}He–^{4}He mixtures with both ^{3}He and ^{4}He superfluid]. For complexity and difficulty, the closest analog in classical physics is perhaps the problem of coupled turbulent velocity and magnetic fields in astrophysical magnetohydrodynamics.

The temperature of the cosmic microwave background radiation is about 2.7 K, and the coldest place found in the Universe is the Boomerang Nebula (≈ 1 K), 5,000 light-years away from Earth in the constellation of Centaurus. Thus, further experimental studies of quantum turbulence, probing physical conditions not known to Nature at temperatures many orders of magnitude lower, may uncover phenomena not yet known to physics.

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