PR

$$e^y(1+x^2)\,dy = 2x(1+e^y)\,dx \quad \left(y' = \frac{2x(1+e^y)}{e^y(1+x^2)} \right)$$

$$\int \frac{e^y}{1+e^y}\,dy = \int \frac{2x}{1+x^2}\,dx$$

$$\ln|1+e^y| = \ln|1+x^2| + C$$

$$e^{\ln|1+e^y|} = e^{\ln|1+x^2|+C} = e^C \cdot e^{\ln(1+x^2)}$$

$$|1+e^y| = K\cdot(1+x^2)$$

$$1+e^y = \pm K(1+x^2)$$

$$1+e^y = A(1+x^2)$$

$$e^y = A(1+x^2)-1$$

$$\boxed{y = \ln(A(1+x^2)-1)}$$

PR $(x^2-1)y' + 2xy^2 = 0 \quad y(0)=1 \qquad y' = \frac{dy}{dx}$

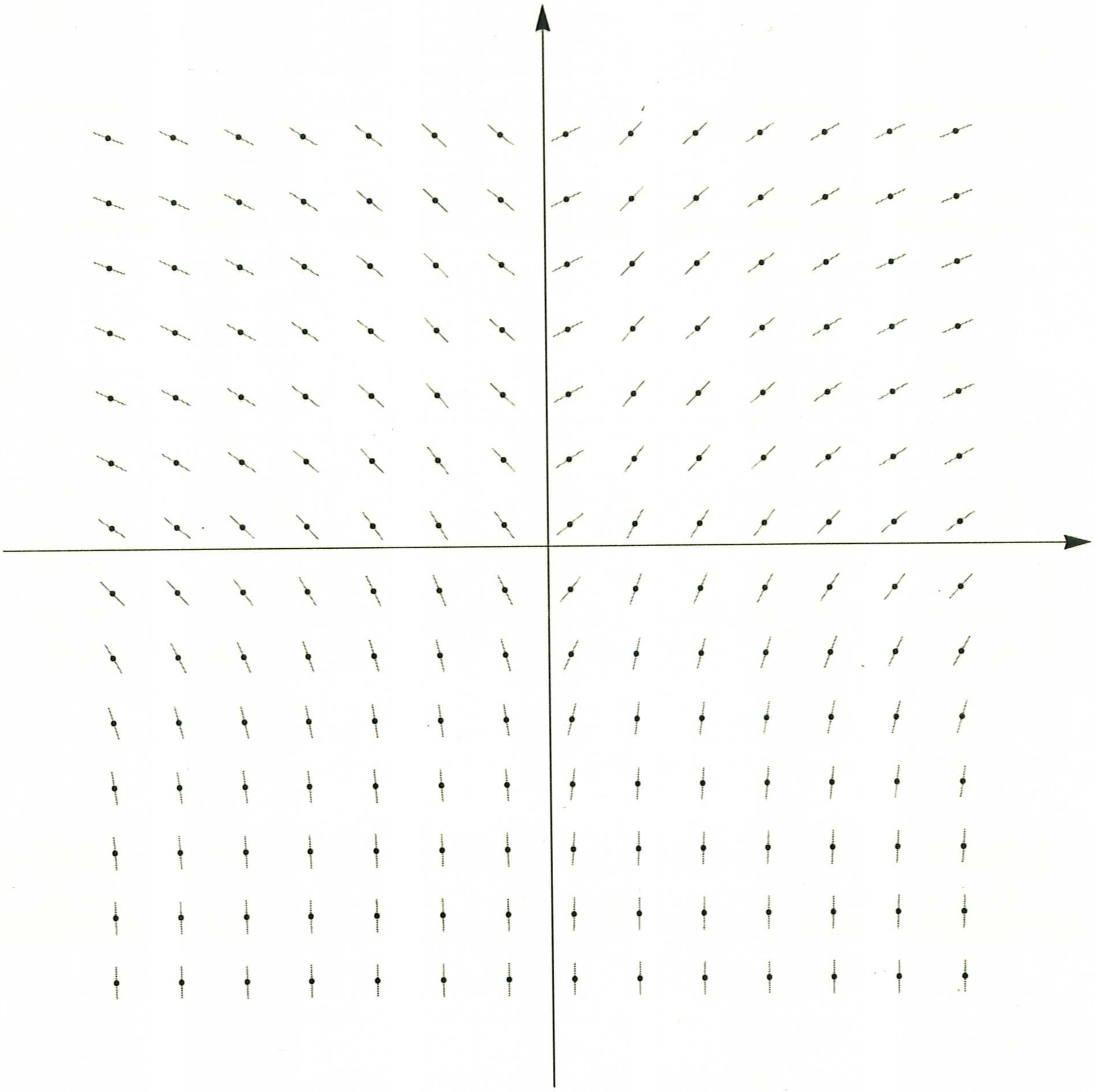
$$\int \frac{dy}{y^2} = \int -\frac{2x}{x^2-1}\,dx$$

$$-\frac{1}{y} = -\ln|x^2-1| + C$$

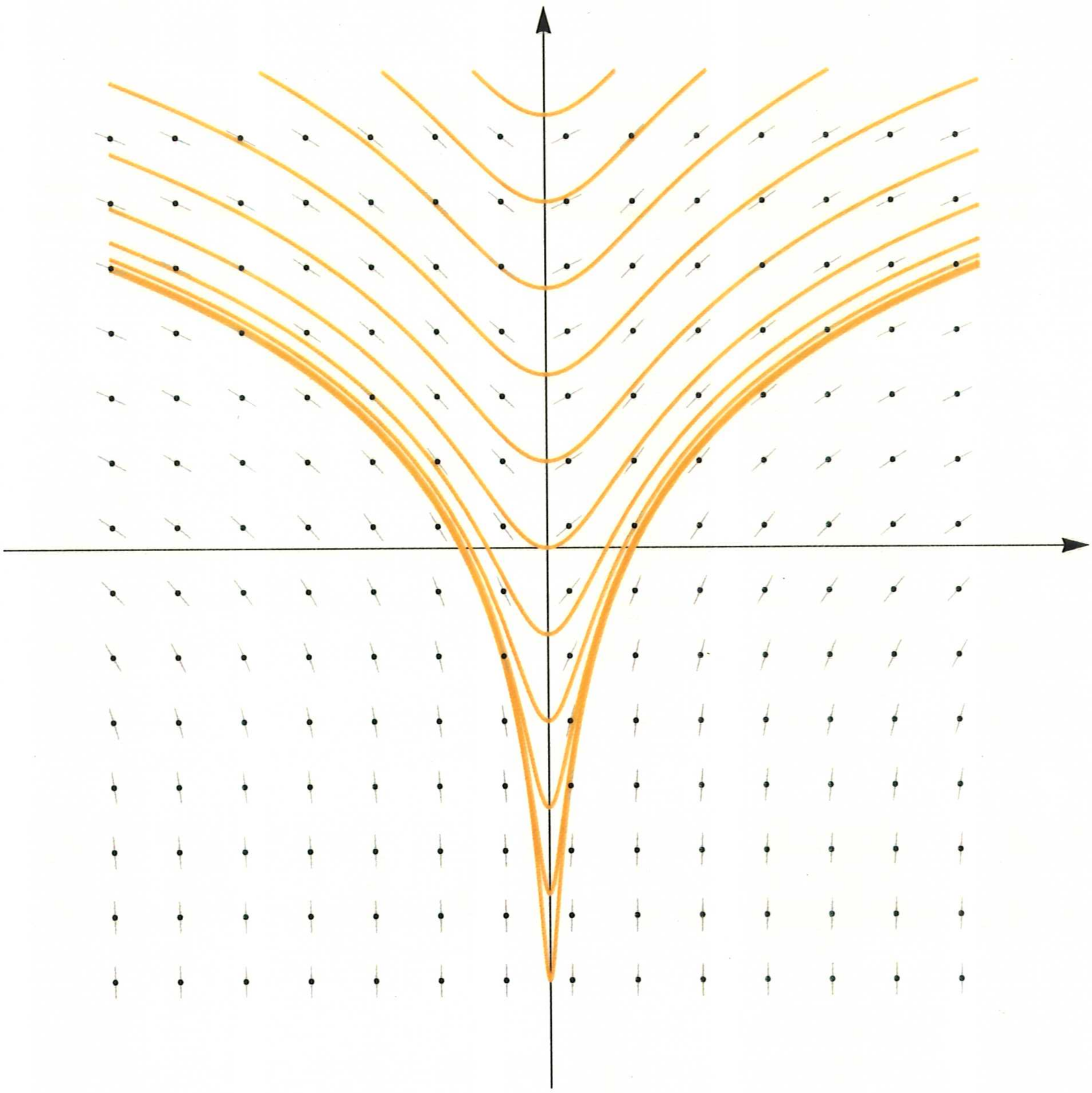
$+y(0)=1$: $-\frac{1}{1} = -\ln|0-1| + C \Rightarrow -1$

$$-\frac{1}{y} = -\ln|x^2-1| - 1 \quad /\cdot(-1)$$

$$\boxed{1 = y(\ln|x^2-1|+1)}$$



Obrázek 1: Směrové pole rovnice $e^y(1+x^2)\mathrm{d}y = 2x(1+e^y)\mathrm{d}x$



Obrázek 2: Integrální křivky rovnice $e^y(1+x^2)\mathrm{d}y = 2x(1+e^y)\mathrm{d}x$

PŘ

$$y' = -\frac{3}{2} + \frac{1}{2(x+y)} = f(x+y)$$

$$u = x + y$$
$$u' = 1 + y' \Rightarrow y' = u' - 1$$

$$u' - 1 = -\frac{3}{2} + \frac{1}{2u}$$

$$u' = -\frac{1}{2} + \frac{1}{2u} = \frac{-u+1}{2u}$$

$$\int \frac{2u}{-u+1}\,du = \int dx$$

$$\int \frac{2u-2+2}{1-u}\,du = \int -2 + \frac{2}{1-u} = -2u - 2\ln|1-u| = x + C$$

$$\Rightarrow -2(x+y) - 2\ln|1-(x+y)| - x = C$$

$$\boxed{-3x - 2y - 2\ln|1-(x+y)| = C}$$

PŘ

$$y' - y = 2x - 3$$
$$y' = y + 2x - 3 = f(y + 2x - 3)$$

$$u = y + 2x - 3$$
$$u' = y' + 2 \Longrightarrow y' = u' - 2$$

$$u' - 2 = u$$
$$u' = u + 2$$

$$\int \frac{du}{u+2} = \int dx$$

$$\ln|u+2| = x + C$$

$$|u+2| = e^{x+C} = e^C \cdot e^x$$

$$u+2 = \mp e^C \cdot e^x$$

$$u = Ae^x - 2$$

$$y + 2x - 3 = Ae^x - 2$$

$$\boxed{y = Ae^x - 2x + 1}$$

$\boxed{\text{Př}}$ $y' = 1 + \sin(x-y)$

substituce: $u = x - y$

$\vdots$

$$\int \frac{du}{\sin u} = -\int dx$$

$$\int \frac{1}{\sin u}\, du = \left\{ \begin{matrix} u = 2t \\ du = 2dt \end{matrix} \right\} = 2\int \frac{dt}{\sin 2t} = \int \frac{1}{\sin t \cos t}\, dt =$$

$$= \left| \frac{\cdot \frac{1}{\cos^2 t}}{\frac{1}{\cos^2 t}} \right\} = \int \frac{\frac{1}{\cos^2 t}}{\operatorname{tg} t}\, dt = \ln|\operatorname{tg} t| + C =$$

$$= \ln\left|\operatorname{tg} \frac{u}{2}\right| + C = -x$$

$$\left|\operatorname{tg} \frac{u}{2}\right| K = e^{-x}$$

$$\operatorname{tg} \frac{u}{2} = Ce^{-x}$$

$$\frac{u}{2} = \operatorname{arctg}(Ce^{-x}) \Rightarrow \boxed{x - y = 2\operatorname{arctg}(Ce^{-x})}$$

HOMOGENNÍ ROVNICE

$y'(x) = f(x,y)$; f - splňuje $\boxed{f(x,y) = f(\alpha x, \alpha y)}$

$\forall (x,y) \in D;\ \alpha \in \mathbb{R}$

- zavádíme substituce

$$\boxed{\begin{pmatrix} y = u \cdot x \\ y' = u'x + u \end{pmatrix}}$$

PŘ

$$y' = \frac{y}{x}\left(1 + \ln \frac{y}{x}\right) = f(x,y)$$

$$f(\alpha x, \alpha y) = \frac{\alpha y}{\alpha x}\left(1 + \ln \frac{\alpha y}{\alpha x}\right) = f(x,y) \checkmark$$

$$y = u \cdot x \quad \left(u = \frac{y}{x}\right)$$

$$y' = \underline{u'x + u = u(1 + \ln u)}$$

$$u'x = u \ln u$$

$$\int \frac{du}{u \ln u} = \int \frac{1}{x}\, dx$$

$$\int \frac{du}{u \ln u} = \left\{ \begin{matrix} \ln u = t \\ \frac{1}{u}\, du = dt \end{matrix} \right\} = \int \frac{1}{t}\, dt = \ln|t| + C =$$

$$= \ln|\ln u| + C$$

$$\ln|\ln u| + \tilde{C} = \ln|x|$$

$$|\ln u| = K \cdot |x|$$

$$\ln u = \mp K|x| = \mp(\mp K x) = Cx \Rightarrow u = e^{Cx}$$

$$u = \frac{y}{x} = e^{cx}$$

$$\boxed{y = xe^{cx}}$$

$\boxed{\text{PŘ}}$

$$y' = \frac{x+y}{x-y} = f(x,y) = f(ax, ay) \quad \forall a \in \mathbb{R} - \{0\}$$

$\rightarrow$ vse je homogeno $\Rightarrow y = ux$

$$y' = u'x + u = \frac{x+ux}{x-ux} = \frac{1+u}{1-u}$$

$$u'x = \frac{1+u}{1-u} - u = \frac{1+u-u+u^2}{1-u} = \frac{1+u^2}{1-u}$$

$$\int \frac{1-u}{1+u^2}\,du = \int \frac{1}{x}\,dx$$

$$\int \frac{1-u}{1+u^2} = \int \frac{1}{1+u^2} + \int \frac{-u}{1+u^2} = \operatorname{arctg} u - \frac{1}{2}\ln|1+u^2| + C$$

$$\operatorname{arctg} u - \frac{1}{2}\ln|1+u^2| + \overset{\ln K}{\underset{\shortparallel}{C}} = \ln|x|$$

$$\operatorname{arctg} u = \ln\left[|x|\sqrt{1+u^2}\cdot K\right]$$

$$\operatorname{arctg} u = \ln\left[|x|\sqrt{1+\left(\frac{y}{x}\right)^2}\cdot K\right] = \ln\left[K\cdot\sqrt{x^2+y^2}\right]$$

$$\boxed{\operatorname{arctg}\left(\frac{y}{x}\right) = \ln\left[K\cdot\sqrt{x^2+y^2}\right]}$$