$\boxed{\text{PŘ}}$

$$e^y(1+x^2)\,dy = 2x(1+e^y)\,dx$$

$$\int \frac{e^y}{1+e^y}\,dy = \int \frac{2x}{1+x^2}\,dx$$

$$\ln|1+e^y| = \ln|1+x^2| + C \qquad a=b \Rightarrow e^a = e^b$$

$$e^{\ln|1+e^y|} = e^{\ln|1+x^2|+C} = e^{\ln|1+x^2|}\,\underbrace{e^C}_{=k} \qquad e^{\ln a} = a$$

$$|1+e^y| = |1+x^2|\cdot k$$

$$1+e^y = k(1+x^2)$$

$$\boxed{y = \ln\left(k(1+x^2)-1\right)}$$

$\boxed{\text{PŘ}}$

$$(x^2-1)y' + 2xy^2 = 0 \qquad y(0)=1 \qquad y' = \frac{dy}{dx}$$

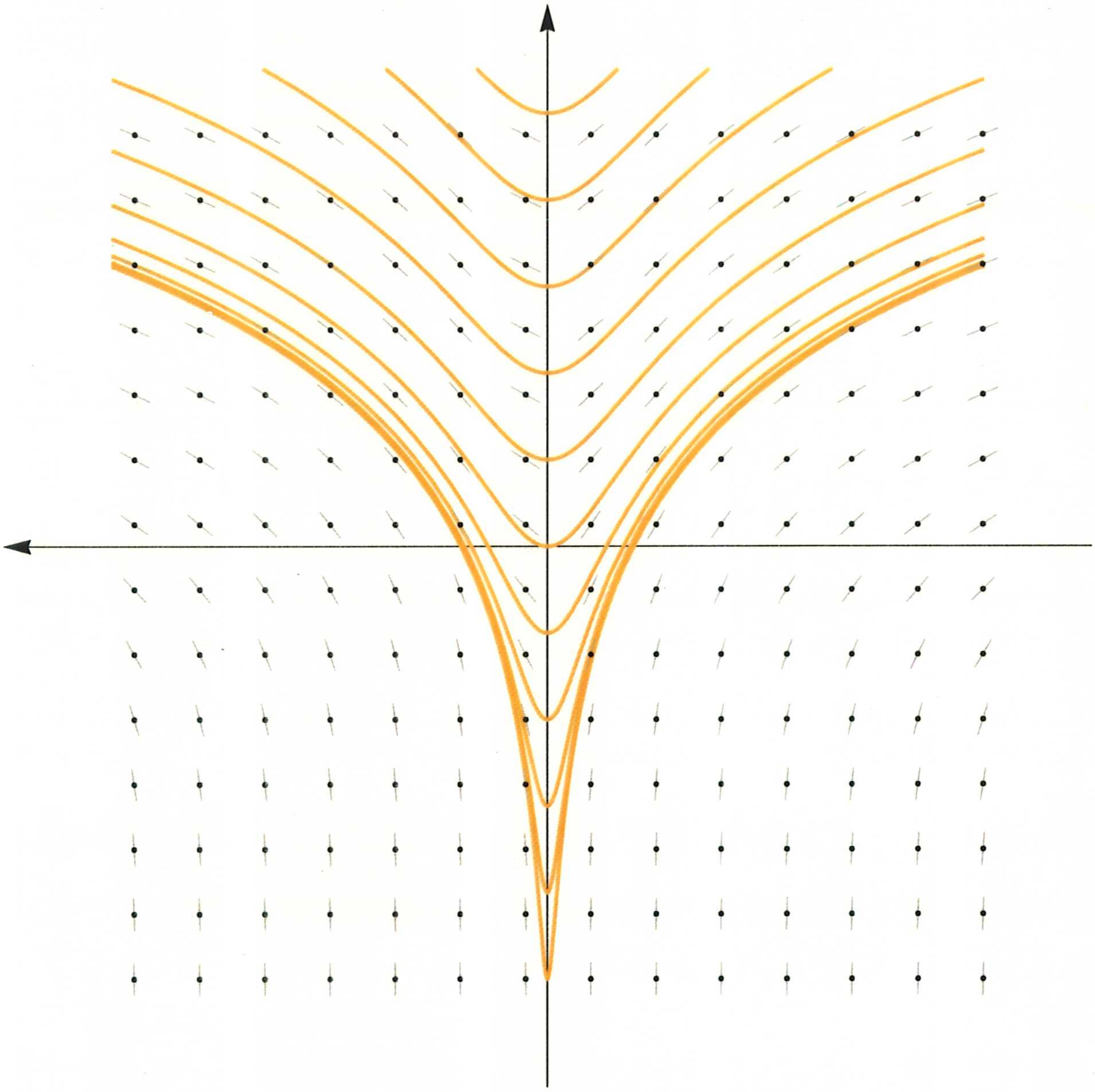
$$(x^2-1)y' = -2xy^2$$

$$\int \frac{dy}{y^2} = \int \frac{-2x}{x^2-1}\,dx$$

$$-\frac{1}{y} = -\ln|x^2-1| + C \rightarrow \text{obecné řešení}$$

$$-\frac{1}{1} = -\ln|0-1| + C \Rightarrow C = -1$$

→ řešení p.ú.: $+\frac{1}{y} = +\ln|x^2-1| + 1$



Obrázek 2: Integrální křivky rovnice $e^y(1 + x^2)dy = 2x(1 + e^y)dx$

ROVNICE PŘEVODITELNÉ NA RCE SE SEPAROVANÝMI PROMĚNNÝMI

- **ROVNICE S PŘÍMKOU**

$$y'(x) = f(ax + by + c)$$

→ substituce $z = ax + by + c$

PŘ

$$(\underbrace{2x - y + 1}_{z})\,dy + (\underbrace{4x - 2y + 6}_{2z+4})\,dx = 0$$

$$z = 2x - y + 1$$

$$dz = 2dx - dy \Rightarrow dy = 2dx - dz$$

$$z(2dx - dz) + (2z + 4)dx = 0$$

$$\cancel{-z\,dz + 4z\,dx}$$

$$2z\,dx - z\,dz + 2z\,dx + 4dx = 0$$

$$(4z + 4)dx = z\,dz$$

$$\int dx = \int \frac{z}{4z+4}\,dz$$

$$4x = \int \frac{z}{z+1}\,dz = \int \frac{z+1-1}{z+1}\,dz = z - \ln|z+1| + C$$

$$\boxed{4x = 2x - y + 1 - \ln|2x - y + 2| + C}$$

(c)

PE

$$y'\sqrt{1+x+y} = x+y-1$$

$$z = 1+x+y$$

a) $dz = dx + dy$ b) $y = z-1-x$

$$y' = z'-1$$

$$(z'-1)\sqrt{z} = z-2 \quad /:\sqrt{z}$$

$$z' = \frac{z-2}{\sqrt{z}} + 1 = \frac{z-2+\sqrt{z}}{\sqrt{z}}$$

$$\int \frac{\sqrt{z}}{z+\sqrt{z}-2}\,dz = \int dx$$

$$\int \frac{\sqrt{z}}{z+\sqrt{z}-2}\,dz = \left\{ \begin{array}{l} \sqrt{z} = t \\ \frac{1}{2\sqrt{z}}\,dz = dt \end{array} \right\} = \int \frac{t \cdot 2t}{t^2+t-2} = 2\int \frac{t^2}{t^2+t-2} =$$

$$= 2\int \frac{t^2+t-2-t+2}{t^2+t-2}\,dt = 2\int 1 + \frac{-t+2}{t^2+t-2}\,dt =$$

$$= 2\int 1 + \frac{4}{3(t-2)} - \frac{1}{3(t-1)}\,dt = 2\left(t + \frac{4}{3}\ln|t-2| - \frac{1}{3}\ln|t-1|\right) + C$$

$$2\left(\sqrt{z} + \frac{4}{3}\ln|\sqrt{z}-2| - \frac{1}{3}\ln|\sqrt{z}-1|\right) + C = x \quad /.\ z \to 1+x+y$$

$y' = \sin(x-y)$

$z = x - y \Rightarrow y = x - z$
$y' = 1 - z'$

$1 - z' = \sin z$

$z' = 1 - \sin z$

$$\int \frac{dz}{1-\sin z} = \int dx$$

$$\int \frac{1}{1-\sin z} = \int \frac{1+\sin z}{1-\sin^2 z} = \int \frac{1}{\cos^2 z} + \int \frac{\sin z}{\cos^2 z} =$$

$$= \operatorname{tg} z + \int \frac{\sin z}{\cos^2 z} = \left\{ \cos z = t \right\} = \operatorname{tg} z + \frac{1}{\cos z} + C$$

$$\boxed{x = \operatorname{tg}(x-y) + \frac{1}{\cos(x-y)} + C}$$

ZE: $0 = \operatorname{tg}(x-y) + \frac{1}{\cos(x-y)} + C - x$ / różniczkujemy

$$\frac{1}{\cos^2(x-y)}(1-y') + \frac{-\sin(x-y)}{\cos^2(x-y)}(1-y') + 0 - 1 = 0$$

wstawiam $y' = \sin(x-y)$

$$\frac{1-\sin(x-y)}{\cos^2(x-y)} + \frac{\sin(x-y)(1-\sin(x-y))}{\cos^2(x-y)} - \frac{\cos^2(x-y)}{\cos^2(x-y)} = 0$$

$$\frac{\sin^2(x-y) - \sin(x-y) + \sin(x-y) - \sin^2(x-y)}{\cos^2(x-y)} = 0 \quad \checkmark$$

HOMOGENNÍ ROVNICE

$$y'(x) = f(x,y) \quad : \quad f(\alpha x, \alpha y) = f(x,y) \quad \forall \alpha$$

substituce: $y = z \cdot x$

PŘ

$$y' = e^{\frac{y}{x}} + \frac{y}{x}$$

$$f(x,y) = e^{\frac{y}{x}} + \frac{y}{x}$$

$$f(\alpha x, \alpha y) = e^{\frac{\alpha y}{\alpha x}} + \frac{\alpha y}{\alpha x} = f(x,y)$$

$$y = z \cdot x$$

$$y' = z'x + z \cdot 1$$

$$z'x + z = e^{z} + z$$

$$z'x = e^{z}$$

$$\int \frac{dz}{e^{z}} = \int \frac{1}{x}\,dx$$

$$-e^{-z} = \ln|x| + C \Rightarrow \boxed{-e^{-\frac{y}{x}} = \ln|x| + C}$$

PŘ

$$y' = \frac{y}{x}\left(1 + \ln\frac{y}{x}\right)$$

$$y = z \cdot x$$

$$z'x + z = z + z\ln z$$

$$\int \frac{dz}{z \ln z} = \int \frac{1}{x}\,dx$$

$$\ln|\ln z| = \ln|x| + C$$

$$e^{\ln|\ln z|} = e^{\ln|x|} \cdot e^{C}$$

$$|\ln z| = |x| \cdot K$$

$$\ln z = \mp K \cdot |x| = \overbrace{\mp(\mp K)}^{A} x$$

$$z = e^{Ax}$$

$$\boxed{y = x e^{Ax}}$$

$\boxed{\text{Př}}$

$$y' = \frac{x+y}{x-y} = f(x,y) = f(\alpha x, \alpha y) \quad \forall x \in \mathbb{R} - \{0\}$$

$$y = ux$$

$$u'x + u = \frac{x+ux}{x-ux} = \frac{1+u}{1-u}$$

$$u'x = \frac{1+u-u+u^2}{1-u} = \frac{1+u^2}{1-u}$$

$$\int \frac{1-u}{1+u^2}\,du = \int \frac{1}{x}\,dx$$

$$\operatorname{arctg} u - \frac{1}{2}\ln|1+u^2| = \ln|x| + C$$

$$\operatorname{arctg} u = \ln\left(|x| \cdot \sqrt{1+u^2}\right) + C$$

$$\operatorname{arctg}\left(\frac{y}{x}\right) = \ln\left(|x|\sqrt{1+\frac{y^2}{x^2}}\right) + C = \ln K$$

$$\boxed{\operatorname{arctg}\left(\frac{y}{x}\right) = \ln\left(K \cdot \sqrt{x^2+y^2}\right)}$$